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深水有限振幅重力波的一种解法*

侯 一 筠
（中国科学院海洋研究所，青岛　266071）

提要　采用变量代换的方法，处理水波的自由边界，获得了新的水波控制方程和边界条件。以摄动法解非线性偏微分方程的近似解析解，求得了与三阶斯托克斯波略有差别的非线性波面和包含振幅的非线性水波色散关系，并且得出了二阶以上的波动势函数在深水情况下不为零的结论。

关键词　有限振幅　摄动法　色散关系

长期以来，求解有限振幅重力波一直是水波动力学研究中的一个难点[1]。困难之处其一在于数学上找不到非线性偏微分方程的解析解，其二是解的边界不能事先固定，而是作为待求的未知函数。因此通过摄动法求其近似解析解是这方面的主要手段之一[3,4]。本文在水波定解问题的基础上，通过变量代换将未知边界变成已知边界，避免了对边值条件进行泰勒展开的传统方法[3,4]。然后以摄动法求解非线性偏微分方程，得出了在准确至三阶时的非线性波面和水波的色散关系。

1. 深水有限振幅重力波的定解问题及变量代换

对所讨论的无旋、深水、二维周期行进波，有控制方程

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial z^2}=0 \qquad (\zeta>z>-\infty) \tag{1.1}$$

自由表面运动学边界条件：

$$\left.\frac{\partial\varphi}{\partial z}\right|_{z=\zeta}=\frac{\partial\zeta}{\partial t}+\frac{\partial\zeta}{\partial x}\left.\frac{\partial\varphi}{\partial x}\right|_{z=\zeta},$$

自由表面动力学边界条件：

$$\left.\frac{\partial\varphi}{\partial t}\right|_{z=\zeta}+g\zeta+\frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial z}\right)^2\right]_{z=\zeta}+\frac{P_s}{\rho}=0$$

此外，对无穷深处还要求$\left.\frac{\partial\varphi}{\partial z}\right|_{z\to-\infty}=0$。式中，$z$ 轴铅直向上为正；φ 表示波动场势函数；ζ 表示自由水面的铅直位移；ρ 表示水密度；P_s 表示自由水面的压强分布，考虑自由水波问题可令 $P_s=0$。

现在做以下变量代换：

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$$\begin{array}{lcl} \xi = x & & x = \xi \\ \eta = z - \zeta & 或 & z = \eta + \zeta \\ \tau = t & & t = \tau \end{array}$$

则有

$$\begin{cases} \dfrac{\partial \varphi}{\partial x} = \dfrac{\partial \varphi}{\partial \xi} - \dfrac{\partial \zeta}{\partial \xi} \dfrac{\partial \varphi}{\partial \eta} \\ \dfrac{\partial \varphi}{\partial z} = \dfrac{\partial \varphi}{\partial \eta} \\ \dfrac{\partial \varphi}{\partial t} = \dfrac{\partial \varphi}{\partial \tau} - \dfrac{\partial \zeta}{\partial \tau} \dfrac{\partial \varphi}{\partial \eta} \\ \dfrac{\partial^2 \varphi}{\partial x^2} = \dfrac{\partial^2 \varphi}{\partial \xi^2} - \dfrac{\partial^2 \zeta}{\partial \xi^2} \dfrac{\partial \varphi}{\partial \eta} - 2 \dfrac{\partial \zeta}{\partial \xi} \dfrac{\partial^2 \varphi}{\partial \xi \partial \eta} + \left(\dfrac{\partial \zeta}{\partial \xi}\right)^2 \dfrac{\partial^2 \varphi}{\partial \eta^2} \\ \dfrac{\partial^2 \varphi}{\partial z^2} = \dfrac{\partial^2 \varphi}{\partial \eta^2} \end{cases} \tag{1.2}$$

将(1.2)代入到控制方程(1.1)和边界条件,就有

$$\begin{cases} \dfrac{\partial^2 \varphi}{\partial \xi^2} + \left[1 + \left(\dfrac{\partial \zeta}{\partial \xi}\right)^2\right] \dfrac{\partial^2 \varphi}{\partial \eta^2} = \dfrac{\partial^2 \zeta}{\partial \xi^2} \dfrac{\partial \varphi}{\partial \eta} + 2 \dfrac{\partial \zeta}{\partial \xi} \dfrac{\partial^2 \varphi}{\partial \xi \partial \eta} \\ \left.\dfrac{\partial \varphi}{\partial \eta}\right|_{\eta \to -\infty} = 0 \\ \left.\dfrac{\partial \varphi}{\partial \eta}\right|_{\eta=0} = \dfrac{\partial \zeta}{\partial \tau} + \dfrac{\partial \zeta}{\partial \xi} \left[\dfrac{\partial \varphi}{\partial \xi} - \dfrac{\partial \zeta}{\partial \xi} \dfrac{\partial \varphi}{\partial \eta}\right]_{\eta=0} \\ \left\{\dfrac{\partial \varphi}{\partial \tau} - \dfrac{\partial \zeta}{\partial \tau} \dfrac{\partial \varphi}{\partial \eta} + \dfrac{1}{2} \left[\left(\dfrac{\partial \varphi}{\partial \xi} - \dfrac{\partial \zeta}{\partial \xi} \dfrac{\partial \varphi}{\partial \eta}\right)^2 + \left(\dfrac{\partial \varphi}{\partial \eta}\right)^2\right]\right\}_{\eta=0} + g\zeta = 0 \end{cases} \tag{1.3}$$

此即在新坐标系下的控制方程和边值条件。在此种数学变换下，求水波的问题就转化成在已知区域 $(-\infty, \infty) \times (-\infty, 0)$ 内求解一个未知函数 $\varphi(\xi, \eta, \tau)$和边界 $\eta = 0$ 上的一个未知函数 $\zeta(\xi, \tau)$。

2. 方程的摄动解

求方程(1.3)的解析解相当困难,对此,以摄动法求其近似解析解,即令

$$\begin{cases} \varphi = \varphi_1 + \varphi_2 + \cdots \\ \zeta = \zeta_1 + \zeta_2 + \cdots \end{cases} \tag{2.1}$$

其中后项是前项的修正量。

将(2.1)代入到(1.3)式的控制方程和边值条件中,就可以得到,

$$一阶: \begin{cases} \dfrac{\partial^2 \varphi_1}{\partial \xi^2} + \dfrac{\partial^2 \varphi_1}{\partial \eta_2} = 0 \\ \left.\dfrac{\partial \varphi_1}{\partial \eta}\right|_{\eta \to -\infty} = 0 \\ \left.\dfrac{\partial \varphi_1}{\partial \eta}\right|_{\eta=0} = \dfrac{\partial \zeta_1}{\partial \tau} \\ \left.\dfrac{\partial \varphi_1}{\partial \tau}\right|_{\eta=0} + g\zeta_1 = 0 \end{cases} \tag{2.2}$$

二阶：
$$\begin{cases}\dfrac{\partial^2\varphi_2}{\partial\xi^2}+\dfrac{\partial^2\varphi_2}{\partial\eta^2}=2\dfrac{\partial\zeta_1}{\partial\xi}\dfrac{\partial\varphi_1}{\partial\xi\,\partial\eta}+\dfrac{\partial^2\zeta_1}{\partial\xi^2}\dfrac{\partial\varphi_1}{\partial\eta}\\ \left.\dfrac{\partial\varphi_2}{\partial\eta}\right|_{\eta=-\infty}=0\\ \left.\dfrac{\partial\varphi_2}{\partial\eta}\right|_{\eta=0}=\dfrac{\partial\zeta_2}{\partial\tau}+\dfrac{\partial\zeta_1}{\partial\xi}\dfrac{\partial\varphi_1}{\partial\xi}\\ \left.\dfrac{\partial\varphi_2}{\partial\tau}\right|_{\eta=0}+g\zeta_2=\left.\dfrac{\partial\zeta_1}{\partial\tau}\dfrac{\partial\varphi_1}{\partial\eta}\right|_{\eta=0}-\dfrac{1}{2}\left[\left(\dfrac{\partial\varphi_1}{\partial\xi}\right)^2+\left(\dfrac{\partial\varphi_1}{\partial\eta}\right)^2\right]_{\eta=0}\end{cases}\tag{2.3}$$

三阶：
$$\begin{cases}\dfrac{\partial^2\varphi_3}{\partial\xi^2}+\dfrac{\partial^2\varphi_3}{\partial\eta^2}=-\dfrac{\partial^2\varphi_1}{\partial\eta^2}\left(\dfrac{\partial\zeta_1}{\partial\xi}\right)^2+2\dfrac{\partial\zeta_1}{\partial\xi}\dfrac{\partial^2\varphi_2}{\partial\xi\,\partial\eta}+2\dfrac{\partial\zeta_2}{\partial\xi}\dfrac{\partial^2\varphi_1}{\partial\xi\,\partial\eta}\\ \qquad+\dfrac{\partial^2\zeta_1}{\partial\xi^2}\dfrac{\partial\varphi_2}{\partial\eta}+\dfrac{\partial^2\zeta_2}{\partial\xi^2}\dfrac{\partial\varphi_1}{\partial\eta}\\ \left.\dfrac{\partial\varphi_3}{\partial\eta}\right|_{\eta\to-\infty}=0\\ \left.\dfrac{\partial\varphi_3}{\partial\eta}\right|_{\eta=0}=\dfrac{\partial\zeta_3}{\partial\tau}+\left[\dfrac{\partial\zeta_1}{\partial\xi}\dfrac{\partial\varphi_2}{\partial\xi}+\dfrac{\partial\zeta_2}{\partial\xi}\dfrac{\partial\varphi_1}{\partial\xi}-\left(\dfrac{\partial\zeta_1}{\partial\xi}\right)^2\dfrac{\partial\varphi_1}{\partial\eta}\right]_{\eta=0}\\ \left.\dfrac{\partial\varphi_3}{\partial\tau}\right|_{\eta=0}+g\zeta_3=\left(\dfrac{\partial\zeta_1}{\partial\tau}\dfrac{\partial\varphi_2}{\partial\eta}+\dfrac{\partial\zeta_2}{\partial\tau}\dfrac{\partial\varphi_1}{\partial\eta}+\dfrac{\partial\zeta_1}{\partial\xi}\dfrac{\partial\varphi_1}{\partial\xi}\dfrac{\partial\varphi_1}{\partial\eta}\right.\\ \qquad\left.-\dfrac{\partial\varphi_1}{\partial\xi}\dfrac{\partial\varphi_2}{\partial\xi}-\dfrac{\partial\varphi_1}{\partial\eta}\dfrac{\partial\varphi_2}{\partial\eta}\right)_{\eta=0}\end{cases}\tag{2.4}$$

… … … …。

一阶解仍然是通常的线性行波

$$\begin{cases}\varphi_1=\dfrac{ag}{\omega}e^{k\eta}\sin(k\xi-\omega\tau)\\ \zeta_1=a\cos(k\xi-\omega\tau)\end{cases}\tag{2.5}$$

由 $\eta=0$ 处的两个边值条件得 $\dfrac{\partial^2\varphi_1}{\partial\tau^2}+g\dfrac{\partial\varphi_1}{\partial\eta}=0$，从而推知在准确至一阶时，色散关系为 $\omega^2=kg$。

将一阶解代入到二阶的控制方程和边值条件中。因现在还不能确定准确至二阶时的色散关系，故在运算中不能使用 $\omega^2=kg$。于是，有

$$\begin{cases}\dfrac{\partial^2\varphi_2}{\partial\xi^2}+\dfrac{\partial^2\varphi_2}{\partial\eta^2}=-\dfrac{3}{2}\dfrac{a^2g}{\omega}k^3e^{k\eta}\sin(k\xi-\omega\tau)\\ \left.\dfrac{\partial\varphi_2}{\partial\eta}\right|_{\eta\to-\infty}=0\\ \left.\dfrac{\partial\varphi_2}{\partial\eta}\right|_{\eta=0}-\dfrac{\partial\zeta_2}{\partial\tau}=-\dfrac{1}{2}a^2k\dfrac{g}{\omega}\sin 2(k\xi-\omega\tau)\\ \left.\dfrac{\partial\varphi_2}{\partial\tau}\right|_{\eta=0}+g\zeta_2=a^2gk\sin^2(k\xi-\omega\tau)-\dfrac{1}{2}\left(\dfrac{agk}{\omega}\right)^2\end{cases}\tag{2.6}$$

解此线性非齐次方程，可得：

$$\begin{cases} \varphi_2 = \dfrac{a^2kg}{\omega}\left(\dfrac{\omega^2 - kg}{-4\omega^2 + 2kg}\right)e^{2k\eta}\sin 2(k\xi - \omega\tau) \\ \qquad + \dfrac{1}{2}a^2k\dfrac{g}{\omega}e^{k\eta}\sin 2(k\xi - \omega\tau) \\ \zeta_2 = \left[2a^2k\left(\dfrac{\omega^2 - kg}{-4\omega^2 + 2kg}\right) + \dfrac{1}{2}a^2k\right]\cos 2(k\xi - \omega\tau) \\ \qquad + \dfrac{1}{2}a^2k\left(1 - \dfrac{kg}{\omega^2}\right) \end{cases} \tag{2.7}$$

因为在准确至二阶时 $\varphi = \varphi_1 + \varphi_2$, $\zeta = \zeta_1 + \zeta_2$, 再把 $\eta = 0$ 处的两个边值条件合并起来,然后将所得到的结果代入,就有

$$(-\omega^2 + kg)\frac{ag}{\omega}\sin(k\xi - \omega\tau) + \left[(\omega^2 - kg)\frac{a^2kg}{\omega}\right.$$
$$+ (-4\omega^2 + kg)\frac{1}{2}a^2k\frac{g}{\omega}$$
$$\left. + \left(a^2gk\omega + \frac{1}{2}\frac{a^2kg^2}{\omega}\right)\right]\sin 2(k\xi - \omega\tau) = 0$$

为使上式恒成立,必须 $\sin(k\xi - \omega\tau)$ 与 $\sin 2(k\xi - \omega\tau)$ 的系数为零。于是就得到了 $\omega^2 = kg$,亦即在准确二阶时不改变色散关系。这和通常的结论是一致的。

因此在准确至二阶时,

$$\begin{cases} \varphi = \dfrac{ag}{\omega}e^{k\eta}\sin(k\xi - \omega\tau) + \dfrac{1}{2}a^2k\dfrac{g}{\omega}e^{k\eta}\sin 2(k\xi - \omega\tau) \\ \zeta = a\cos(k\xi - \omega\tau) + \dfrac{1}{2}a^2k\cos 2(k\xi - \omega\tau) \end{cases} \tag{2.8}$$

新坐标系下 φ 的二阶解不再是零解,由此可预测三阶以上的解也不再是零解[3,4]。另外本文所求出的波面 ζ 与二阶斯托克斯波[5]相比缺少常数项 $\frac{1}{2}a^2k$,这是因为在我们的问题里 $\eta = 0$ 总是代表平均水面。

在计算三阶解里,注意到由于 $\varphi_1, \zeta_1, \varphi_2, \zeta_2$ 的作用非齐项中所出现的一次谐波将会导致波解产生长期项,使正则摄动法失效。对此,一般有两种处理方法,一是把频率 ω 亦按小参数展开[3],另一是采用多重尺度方法[2]。本文将采用第一种方法,而把后一种方法留给以后讨论振幅 a 是变量的情形。

对前面的结果分析得知,摄动法所依赖的小参数是 $\varepsilon = ak$, 即波陡。于是可将色散关系表示为

$$\omega^2 = kg[\sigma_0 + \sigma_1(ak) + \sigma_2(ak)^2 + \cdots] \tag{2.9}$$

由前两阶的结果已知 $\sigma_0 = 1$, $\sigma_1 = 0$,故

$$\omega^2 = kg[1 + \sigma_2(ak)^2 + o(a^2k^2)]$$

在准确至三阶时就可以求出 σ_2, φ_3 及 ζ_3。

将已求得的 $\varphi_1, \zeta_1, \varphi_2, \zeta_2$ 代入三阶控制方程和边值条件并在计算中使用上式,就有

$$\begin{cases}\dfrac{\partial^2\varphi_3}{\partial\xi^2}+\dfrac{\partial^2\varphi_3}{\partial\eta^2}=-3a^{3'}k^3\omega e^{k\eta}\sin 3(k\xi-\omega\tau)\\ \left.\dfrac{\partial\varphi_3}{\partial\eta}\right|_{\eta\to-\infty}=0\\ \left.\dfrac{\partial\varphi_3}{\partial\eta}\right|_{\eta=0}-\dfrac{\partial\zeta_3}{\partial\tau}=-\dfrac{3}{4}a^3k^2\omega\sin 3(k\xi-\omega\tau)\\ \qquad\qquad+\left(\sigma_2-\dfrac{3}{4}\right)a^3k^2\sqrt{kg}\sin(k\xi-\omega\tau)\\ \left.\dfrac{\partial\varphi_3}{\partial\tau}\right|_{\eta}+g\zeta_3=-\dfrac{3}{4}a^3k\omega^2\cos 3(k\xi-\omega\tau)-\dfrac{1}{4}a^3k^2g\cos(k\xi-\omega\tau)\end{cases}\tag{2.10}$$

将后两个边值条件合并可得

$$\left[\frac{\partial^2\varphi_3}{\partial\tau^2}+g\frac{\partial\varphi}{\partial\eta}\right]_{\eta=0}=-3a^3k\omega^3\sin 3(k\xi-\omega\tau)$$
$$+(\sigma_2-1)a^3k(kg)^{3/2}\sin(k\xi-\omega\tau)$$

三阶解可以表示为一次谐波项与三次谐波项的迭加，即 $\varphi_3=\varphi_3^{(1)}+\varphi_3^{(3)},\zeta_3=\zeta_3^{(1)}+\zeta_3^{(3)}$。方程(2.10)化成

$$\begin{cases}\dfrac{\partial^2\varphi_3^{(1)}}{\partial\xi^2}+\dfrac{\partial^2\varphi_3^{(1)}}{\partial\eta^2}=0\\ \left.\dfrac{\partial\varphi_3^{(1)}}{\partial\eta}\right|_{\eta\to-\infty}=0\\ \left[\dfrac{\partial^2\varphi_3^{(1)}}{\partial\tau^2}+g\dfrac{\partial\varphi_3^{(1)}}{\partial\eta}\right]_{\eta=0}=(\sigma_2-1)a^3k(kg)^{3/2}\sin(k\xi-\omega\tau)\end{cases}\tag{2.11}$$

与

$$\begin{cases}\dfrac{\partial^2\varphi_3^{(3)}}{\partial\xi^2}+\dfrac{\partial^2\varphi_3^{(3)}}{\partial\eta^2}=-3a^3k^3\omega e^{k\eta}\sin 3(k\xi-\omega\tau)\\ \left.\dfrac{\partial\varphi_3^{(3)}}{\partial\eta}\right|_{\eta\to-\infty}=0\\ \left[\dfrac{\partial^2\varphi_3^{(3)}}{\partial\tau^2}+g\dfrac{\partial\varphi_3^{(3)}}{\partial\eta}\right]_{\eta=0}=-3a^3k\omega^3\sin 3(k\xi-\omega\tau)\end{cases}\tag{2.12}$$

为使(2.11)中的长期项消失，必须 $\sigma_2=1$，则

$$\begin{cases}\varphi_3^{(1)}=0\\ \zeta_3^{(1)}=-\dfrac{1}{4}a^3k^2\cos(k\xi-\omega\tau)\end{cases}\tag{2.13}$$

准确至三阶时色散关系为

$$\omega^2=kg[1+a^2k^2]\tag{2.14}$$

这也和通常的结论是一致的。

最后求解线性非齐次方程(2.12)可得

$$\begin{cases}\varphi_3^{(3)}=\dfrac{3}{8}a^3k^2\dfrac{g}{\omega}e^{k\eta}\sin 3(k\xi-\omega\tau)\\ \zeta_3^{(3)}=\dfrac{3}{8}a^3k^2\cos 3(k\xi-\omega\tau)\end{cases}\tag{2.15}$$

因此在准确至三阶时

$$\begin{cases} \varphi = \dfrac{ag}{\omega} e^{k\eta} \sin(k\xi - \omega\tau) + \dfrac{1}{2} a^2 k \dfrac{g}{\omega} e^{k\eta} \sin 2(k\xi - \omega\tau) \\ \qquad + \dfrac{3}{8} a^3 k^2 \dfrac{g}{\omega} e^{k\eta} \sin 3(k\xi - \omega\tau) \\ \zeta = \left(a - \dfrac{1}{4} a^3 k^2\right) \cos(k\xi - \omega\tau) + \dfrac{1}{2} a^2 k \cos 2(k\xi - \omega\tau) \\ \qquad + \dfrac{3}{8} a^3 k^2 \cos 3(k\xi - \omega\tau) \end{cases} \tag{2.16}$$

与斯托克斯波[5]相比，$\cos(k\xi - \omega\tau)$ 的系数不一致，这是本文的变量代换方法与展开了 $\frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2\right]$ 计算到三阶近似的缘故，因此我们的结果显得更合理一些。

3. 结语

采用恰当的变量代换处理水波的自由边界是一个合理的方法，因由此转换而导出的新的定解问题具有数学上的严密性，代替了对边值条件进行泰勒展开的传统方法。本文的结果恰好说明以摄动法求解水波问题这两种处理方式是一致的。但是，对于研究调制波、定形波以及非线性色散关系的解析形式，将会体现出本文的优越性，因为泰勒展开不能保留无穷多项。

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A SOLUTION OF THE GRAVITY WAVE OF FINITE AMPLITUDE IN DEEP WATER*

Hou Yijun

(Institute of Oceanology, Academia Sinica, Qingdao 266071)

ABSTRACT

Using, $\xi = x$ or $x = \xi$

$\eta = z - \zeta$ $\quad z = \eta + \zeta$

$\tau = t$ $\quad t = \tau$,

the method of variable substitution in the water-wave equation and boundary conditions, this paper deals with the free-surface of the water-wave. The changed water-wave equation and boundary conditions may be written in the form

$$\frac{\partial^2\varphi}{\partial\xi^2} + \left[1 + \left(\frac{\partial\zeta}{\partial\xi}\right)^2\right]\frac{\partial^2\varphi}{\partial\eta^2} = \frac{\partial^2\zeta}{\partial\xi^2}\frac{\partial\varphi}{\partial\eta} + 2\frac{\partial\zeta}{\partial\xi}\frac{\partial^2\varphi}{\partial\xi\partial\eta}$$

$$\left.\frac{\partial\varphi}{\partial\eta}\right|_{\eta\to-\infty} = 0$$

$$\left.\frac{\partial\varphi}{\partial\eta}\right|_{\eta=0} = \frac{\partial\zeta}{\partial\tau} + \frac{\partial\zeta}{\partial\xi}\left[\frac{\partial\varphi}{\partial\xi} - \frac{\partial\zeta}{\partial\xi}\frac{\partial\varphi}{\partial\eta}\right]_{\eta=0}$$

$$\left\{\frac{\partial\varphi}{\partial\tau} - \frac{\partial\zeta}{\partial\tau}\frac{\partial\varphi}{\partial\eta} + \frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial\xi} - \frac{\partial\zeta}{\partial\xi}\frac{\partial\varphi}{\partial\eta}\right)^2 + \left(\frac{\partial\varphi}{\partial\eta}\right)^2\right]\right\}_{\eta=0} + g\zeta = 0。$$

The asymptotic method is applicable to the solution of nonliner wave. Third order STOKES wave and nonliner dispersion relation are solved. A new conclusion is derived that the high order solution of the potential function is non-zero in deep water.

Key words Finite Amplitude, Asymptotic Method, Dispersion Relation.

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