

# 浅水波方程的 TVD 有限差分数值模拟\*

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**摘要** 本文将 Harten 的总变差缩减有限差分格式 (TVD), 应用于非线性的不可压缩的浅水波方程数值求解。通过对一维、二维溃坝洪水波的数值模拟, 成功地模拟了溃坝问题中的激波和膨胀波现象, 保持了解在间断处陡峭, 而在其两侧无振动。

Harten 的双曲型守恒方程的二阶精度 TVD 差分格式, 自 1983 年提出后, 受到高度重视和广泛应用。它在数值模拟可压缩气体动力学的激波现象中, 成功地显示出其高精度、高分辨率和稳定性等优点<sup>1)</sup>。本文基于 Harten 的 TVD 差分思想, 通过运用一维、二维的守恒型浅水波方程的拟线性形式, 给出了数值差分格式。

溃坝问题的数值模拟, 一直是计算流体动力学的一个重要课题, 具有一定的理论意义和实际意义, 它不仅涉及到数值模拟非线性偏微分方程的间断解问题; 而且可为人们预防洪水灾害提供预测。已有文献[1],[2],[4]作过此方面的探讨, 但都存在差分格式精度较低、激波区有色散效应或耗散性过强等问题, 本文通过对瞬溃坝洪水间断波在坝下游河道中的演进模拟, 以说明本差分格式对浅水波演进模拟的优良性能。

## 一、一维浅水波方程的差分计算格式

根据流场中的水深及流速, 给出一维浅水波方程守恒形式如下:

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = H(U) \quad (1.1a)$$

式中

$$U = \begin{pmatrix} \eta \\ u \end{pmatrix}, \quad F(U) = \begin{pmatrix} \eta u \\ \frac{1}{2} u^2 + g\eta \end{pmatrix},$$
$$H(U) = \begin{pmatrix} 0 \\ -\left(R(u) + g \frac{\partial h}{\partial x}\right) \end{pmatrix} \quad (1.1b)$$

式中,  $\eta$  为水深;  $u$  为  $x$  轴(沿河流方向)正向上的流速(见图 1);  $R(u)$  为  $x$  方向上的运动

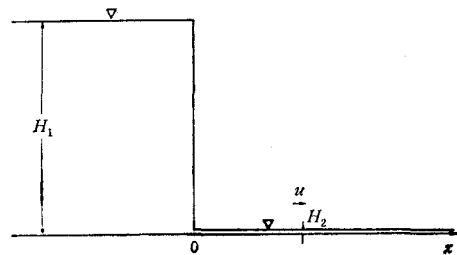


图 1 一维溃坝的初始条件 ( $H_1 = 1, H_2 = 0.01, u = 0$ )

Fig. 1 The initial conditions of one-dimensional dam-break, ( $H_1 = 1, H_2 = 0.01, u = 0$ )

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1) 王如云、刘儒勋, 1988。二维冲击波在多重障碍物中的传播数值模拟。

粘滯力;  $\frac{\partial h}{\partial x}$  为河床沿  $x$  轴方向上的变化梯度。

对应 (1.1) 中的  $\frac{dF}{dU}$  的特征值为  $u + \sqrt{g\eta}$ ,  $u - \sqrt{g\eta}$ , 而对应各特征值的右特征向量为:

$$\begin{pmatrix} 1 \\ u + \sqrt{g\eta} \end{pmatrix}, \begin{pmatrix} 1 \\ u - \sqrt{g\eta} \end{pmatrix}$$

对应各特征值的左特征向量为:

$$\frac{1}{2\sqrt{g\eta}}(-u + \sqrt{g\eta}, 1), \frac{1}{2\sqrt{g\eta}}(u + \sqrt{g\eta}, -1)$$

依据文献[3]给出一维浅水波方程的有限差分格式如下:

$$\hat{u}_{i+\frac{1}{2}} = \begin{cases} u_{i+1} & (d=0) \\ u_i & (d=\infty) \\ \frac{d \cdot u_i + u_{i+1}}{1+d} & (0 < d < \infty) \end{cases} \quad \hat{\eta}_{i+\frac{1}{2}} = \begin{cases} \eta_{i+1} & (d=0) \\ \eta_i & (d=\infty) \\ \frac{d \cdot \eta_i + \eta_{i+1}}{1+d} & (0 < d < \infty) \end{cases} \quad (1.2)$$

其中,

$$d = \sqrt{\frac{\eta_i}{\eta_{i+1}}}$$

$$\begin{cases} \bar{a}_{i+\frac{1}{2}}^1 = \hat{u}_{i+\frac{1}{2}} - \sqrt{g\hat{\eta}_{i+\frac{1}{2}}} \\ \bar{a}_{i+\frac{1}{2}}^2 = \hat{u}_{i+\frac{1}{2}} + \sqrt{g\hat{\eta}_{i+\frac{1}{2}}} \end{cases} \quad (1.3a)$$

$$\quad (1.3b)$$

$$\begin{cases} [\eta]^* = \eta_{i+1} - \eta_i \\ [\eta u]^* = (\eta u)_{i+1} - (\eta u)_i \end{cases} \quad (1.4a)$$

$$\quad (1.4b)$$

$$\alpha_{i+\frac{1}{2}}^1 = \begin{cases} \frac{1}{2} \sqrt{\eta_{i+1}/g} (\bar{a}_{i+\frac{1}{2}}^2 - u_{i+1}) & (d=0) \\ \frac{1}{2} \sqrt{\eta_i/g} (u_i - \bar{a}_{i+\frac{1}{2}}^1) & (d=\infty) \\ \frac{1}{2} \frac{1}{\sqrt{g\hat{\eta}_{i+\frac{1}{2}}}} (\bar{a}_{i+\frac{1}{2}}^2 [\eta]^* - [\eta u]^*) & (0 < d < \infty) \end{cases} \quad (1.5a)$$

$$\alpha_{i+\frac{1}{2}}^2 = \begin{cases} \frac{1}{2} \sqrt{\eta_{i+1}/g} (u_{i+1} - \bar{a}_{i+\frac{1}{2}}^1) & (d=0) \\ \frac{1}{2} \sqrt{\eta_i/g} (\bar{a}_{i+\frac{1}{2}}^1 - u_i) & (d=\infty) \\ \frac{1}{2} \frac{1}{\sqrt{g\hat{\eta}_{i+\frac{1}{2}}}} (-\bar{a}_{i+\frac{1}{2}}^1 [\eta]^* + [\eta u]^*) & (0 < d < \infty) \end{cases} \quad (1.5b)$$

$$Q(x) = \begin{cases} \frac{x^2}{4\varepsilon} + \varepsilon & |x| < z\varepsilon \\ |x| & |x| \geq z\varepsilon \end{cases} \quad (1.6)$$

一般取  $\varepsilon = 0.1$ 。

$$\lambda = \frac{\Delta t}{\Delta x} \quad (1.7)$$

$$v_{i+\frac{1}{2}}^k = \lambda \bar{a}_{i+\frac{1}{2}}^k \quad (k = 1, 2) \quad (1.8)$$

$$\tilde{g}_{i+\frac{1}{2}}^k = \frac{1}{2} [Q(v_{i+\frac{1}{2}}^k) - (v_{i+\frac{1}{2}}^k)^2] \cdot \alpha_{i+\frac{1}{2}}^k \quad (k = 1, 2) \quad (1.9)$$

$$s_{i+\frac{1}{2}}^k = \text{sgn}(\tilde{g}_{i+\frac{1}{2}}^k) \quad (k = 1, 2) \quad (1.10)$$

$$g_i^k = s_{i+\frac{1}{2}}^k \cdot \max(0, \min(|\tilde{g}_{i+\frac{1}{2}}^k|, s_{i+\frac{1}{2}}^k \cdot \tilde{g}_{i-\frac{1}{2}}^k)) \quad (k = 1, 2) \quad (1.11)$$

$$r_{i+\frac{1}{2}}^k = \begin{cases} (g_{i+1}^k - g_i^k) / \alpha_{i+\frac{1}{2}}^k & \alpha_{i+\frac{1}{2}}^k \neq 0 \\ 0 & \alpha_{i+\frac{1}{2}}^k = 0 \end{cases} \quad (k = 1, 2) \quad (1.12)$$

$$R_{i+\frac{1}{2}}^1 = \begin{pmatrix} 1 \\ \bar{a}_{i+\frac{1}{2}}^1 \end{pmatrix}, \quad R_{i+\frac{1}{2}}^2 = \begin{pmatrix} 1 \\ \bar{a}_{i+\frac{1}{2}}^2 \end{pmatrix} \quad (1.13)$$

$$\begin{aligned} \tilde{f}_{i+\frac{1}{2}}^n &= \frac{1}{2} [F(U_i^n) + F(U_{i+1}^n)] + \frac{1}{2\lambda} \cdot \sum_{k=1}^2 R_{i+\frac{1}{2}}^k \cdot [g_i^k + g_{i+1}^k - Q(v_{i+\frac{1}{2}}^k + r_{i+\frac{1}{2}}^k) \alpha_{i+\frac{1}{2}}^k] \\ &\quad \cdot [g_i^k + g_{i+1}^k - Q(v_{i+\frac{1}{2}}^k + r_{i+\frac{1}{2}}^k) \alpha_{i+\frac{1}{2}}^k] \end{aligned} \quad (1.14)$$

$$\tilde{U}_i = U_i^n - \lambda (\tilde{f}_{i+\frac{1}{2}}^n - \tilde{f}_{i-\frac{1}{2}}^n) \quad (1.15)$$

$$U_i^{n+1} = \tilde{U}_i + \Delta t \cdot H(U_i^n) \quad (1.16)$$

## 二、二维浅水波方程的差分计算格式

首先, 考虑流场中的水深及水平面上直角坐标系下的流速两个分量, 给出二维浅水波方程守恒形式如下:

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} + \frac{\partial G(U)}{\partial y} = H(U) \quad (2.1a)$$

式中,

$$\begin{aligned} U &= \begin{pmatrix} \eta \\ u \\ v \end{pmatrix}, \quad F(U) = \begin{pmatrix} \eta u \\ \frac{1}{2} u^2 + g\eta \\ uv \end{pmatrix} \\ G(U) &= \begin{pmatrix} \eta v \\ uv \\ \frac{1}{2} v^2 + g\eta \end{pmatrix}, \quad H(U) = - \begin{pmatrix} 0 \\ R_x + gh_x \\ R_y + gh_y \end{pmatrix} \end{aligned} \quad (2.1b)$$

式中,  $\eta$  为水深;  $u, v$  为  $x, y$  轴方向上的流速分量;  $R_x, R_y$  表示  $x, y$  轴上的运动粘滞力;  $h_x, h_y$  表示  $x, y$  轴上河床的变化梯度。

下面给出(2.1)的具体差分格式:

$$U^{n+1} = L_y \cdot L_x U^n + \Delta t_1 H(U^n) \quad (2.2a)$$

$$U^{n+2} = L_x \cdot L_y U^{n+1} + \Delta t_2 H(U^{n+1}) \quad (2.2b)$$

式中,  $L_x, L_y$  分别是  $\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = 0$ ,  $\frac{\partial U}{\partial t} + \frac{\partial G(U)}{\partial y} = 0$  的 TVD 差分算

子。

$\Delta t_1$  在全流场满足:

$$\begin{cases} \lambda_x \cdot \max_i \{ |\bar{a}_{i+\frac{1}{2}}^k(U^n)| \} \leq \mu \\ \lambda_y \cdot \max_j \{ |\bar{b}_{j+\frac{1}{2}}^k(U^n)| \} \leq \mu \end{cases} \quad (1 \leq k \leq 3) \quad (2.3)$$

$\Delta t_2$  在全流场满足:

$$\begin{cases} \lambda_x \cdot \max_i \{ |\bar{a}_{i+\frac{1}{2}}^k(U^{n+1})| \} \leq \mu \\ \lambda_y \cdot \max_j \{ |\bar{b}_{j+\frac{1}{2}}^k(U^{n+1})| \} \leq \mu \end{cases} \quad (1 \leq k \leq 5) \quad (2.4)$$

式中,  $\lambda_x = \frac{\Delta t_1}{\Delta x}$  [式(2.3)] 或  $\lambda_x = \frac{\Delta t_2}{\Delta x}$  [式(2.4)],  $\lambda_y = \frac{\Delta t_1}{\Delta y}$  [式(2.3)] 或  $\lambda_y = \frac{\Delta t_2}{\Delta y}$  [式(2.4)],  $\mu$  为小于等于 1 的正常数, 一般取 0.95。  $\bar{a}^k, \bar{b}^k$  为  $\frac{dF}{dU}, \frac{dG}{dU}$  的特征值。

下面先给出  $L_x$  的算法。

对应(2.1)中的  $\frac{dF}{dU}$  的特征值为  $u - \sqrt{g\eta}, u, u + \sqrt{g\eta}$ 。 对应各特征值的右特征向量为:

$$\left(1, -\sqrt{\frac{g}{\eta}}, \frac{v}{\eta}\right)^T, (0, 0, 1)^T, \left(1, \sqrt{\frac{g}{\eta}}, \frac{v}{\eta}\right)^T$$

对应各特征值的左特征向量为:

$$\left(1, -\sqrt{\frac{\eta}{g}}, 0\right), \left(-\frac{v}{\eta}, 0, 1\right), \left(1, \sqrt{\frac{\eta}{g}}, 0\right)$$

依据文献[3], 有:

$$\begin{aligned} U_{ij}^{n+\frac{1}{2}} = L_x(U^n) = & U_{ij}^n - \lambda_x [\bar{f}_{i+\frac{1}{2},j}(U_{i-1,j}^n, U_{i,j}^n, U_{i+1,j}^n, U_{i+2,j}^n) \\ & - \bar{f}_{i-\frac{1}{2},j}(U_{i-2,j}^n, U_{i-1,j}^n, U_{i,j}^n, U_{i+1,j}^n)] \end{aligned} \quad (2.5)$$

$$\begin{aligned} \bar{f}_{i+\frac{1}{2},j}(U_{i-1,j}^n, U_{i,j}^n, U_{i+1,j}^n, U_{i+2,j}^n) = & \frac{1}{2} [F(U_{i,j}^n) + F(U_{i+1,j}^n)] \\ & + \frac{1}{2\lambda_x} \sum_{k=1}^3 R_{i+\frac{1}{2},j}^k \cdot \beta_{i+\frac{1}{2},j}^k \end{aligned} \quad (2.6)$$

$$\begin{aligned} \beta_{i+\frac{1}{2},j}^k = & \theta_{i+\frac{1}{2},j}^k \cdot (g_{i,j}^k + g_{i+1,j}^k) \\ & - Q(v_{x,i+\frac{1}{2},j}^k + \theta_{i+\frac{1}{2},j}^k \cdot \gamma_{i+\frac{1}{2},j}^k) \cdot \alpha_{i+\frac{1}{2},j}^k \end{aligned} \quad (2.7)$$

$$\lambda_x = \frac{\Delta t_1}{\Delta x} \quad (2.8)$$

$$R_{i+\frac{1}{2},j}^1 = \begin{pmatrix} 1 \\ -\sqrt{\frac{g}{\eta_{i+\frac{1}{2},j}}} \\ \frac{\theta_{i+\frac{1}{2},j}}{\eta_{i+\frac{1}{2},j}} \end{pmatrix}, R_{i+\frac{1}{2},j}^2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, R_{i+\frac{1}{2},j}^3 = \begin{pmatrix} 1 \\ \sqrt{\frac{g}{\eta_{i+\frac{1}{2},j}}} \\ \frac{\theta_{i+\frac{1}{2},j}}{\eta_{i+\frac{1}{2},j}} \end{pmatrix} \quad (2.9)$$

$$Q(x) = \begin{cases} \frac{x^2}{4\varepsilon} + \varepsilon & |x| < 2\varepsilon \\ |x| & |x| \geq 2\varepsilon \end{cases} \quad (\text{取 } \varepsilon = 0.1) \quad (2.10)$$

$$v_{i+\frac{1}{2},i}^k = \lambda \bar{a}_{i+\frac{1}{2},i}^k \quad (2.11)$$

$$\alpha_{i+\frac{1}{2},i}^k = L_{i+\frac{1}{2},i}^k \cdot \Delta_{i+\frac{1}{2}} U_{i,i}^n \quad (2.12)$$

$$L_{i+\frac{1}{2},i}^1 = \left( 1, -\sqrt{\frac{\hat{\eta}_{i+\frac{1}{2},i}}{g}}, 0 \right) \quad (2.13a)$$

$$L_{i+\frac{1}{2},i}^2 = \left( -\frac{\vartheta_{i+\frac{1}{2},i}}{\hat{\eta}_{i+\frac{1}{2},i}}, 0, 1 \right) \quad (2.13b)$$

$$L_{i+\frac{1}{2},i}^3 = \left( 1, \sqrt{\frac{\hat{\eta}_{i+\frac{1}{2},i}}{g}}, 0 \right) \quad (2.13c)$$

$$\theta_{i,i}^k = \begin{cases} |\alpha_{i+\frac{1}{2},i}^k - \alpha_{i-\frac{1}{2},i}^k| / (|\alpha_{i+\frac{1}{2},i}^k| + |\alpha_{i-\frac{1}{2},i}^k|) & (\alpha_{i\pm\frac{1}{2},i}^k \text{ 不都为零}) \\ 0 & (\alpha_{i\pm\frac{1}{2},i}^k \text{ 都为零}) \end{cases} \quad (2.14)$$

$$\theta_{i+\frac{1}{2},i}^k = 1 + 2\max\{\theta_{i,i}^k, \theta_{i+1,i}^k\} \quad (2.15)$$

$$\tilde{g}_{i+\frac{1}{2},i}^k = \frac{1}{2} [Q(v_{i+\frac{1}{2},i}^k) - (v_{i+\frac{1}{2},i}^k)^2] \cdot \alpha_{i+\frac{1}{2},i}^k \quad (2.16)$$

$$s_{i+\frac{1}{2},i}^k = \text{sgn}(\tilde{g}_{i+\frac{1}{2},i}^k) \quad (2.17)$$

$$g_{i,i}^k = s_{i+\frac{1}{2},i}^k \cdot \max\{0, \min(|\tilde{g}_{i+\frac{1}{2},i}^k|, s_{i+\frac{1}{2},i}^k \cdot \tilde{g}_{i-\frac{1}{2},i}^k)\} \quad (2.18)$$

$$r_{i+\frac{1}{2},i}^k = \begin{cases} (g_{i+1,i}^k - g_{i,i}^k) / \alpha_{i+\frac{1}{2},i}^k & \alpha_{i+\frac{1}{2},i}^k \neq 0 \\ 0 & \alpha_{i+\frac{1}{2},i}^k = 0 \end{cases} \quad (2.19)$$

取  $d = \sqrt{\frac{\eta_{i,i}}{\eta_{i+1,i}}}$

$$\hat{\eta}_{i+\frac{1}{2},i} = \frac{d \cdot \eta_{i,i} + \eta_{i+1,i}}{1 + d} \quad (2.20a)$$

$$\hat{u}_{i+\frac{1}{2},i} = \frac{d \cdot u_{i,i} + u_{i+1,i}}{1 + d} \quad (2.20b)$$

$$\hat{\vartheta}_{i+\frac{1}{2},i} = \frac{d \cdot v_{i,i} + v_{i+1,i}}{1 + d} \quad (2.20c)$$

$$\bar{a}_{i+\frac{1}{2},i}^1 = \hat{u}_{i+\frac{1}{2},i} - \sqrt{g\hat{\eta}_{i+\frac{1}{2},i}} \quad (2.21a)$$

$$\bar{a}_{i+\frac{1}{2},i}^2 = \hat{u}_{i+\frac{1}{2},i} \quad (2.21b)$$

$$\bar{a}_{i+\frac{1}{2},i}^3 = \hat{u}_{i+\frac{1}{2},i} + \sqrt{g\hat{\eta}_{i+\frac{1}{2},i}} \quad (2.21c)$$

注意: 若  $\eta_{i+1,i} = 0$ , 则取  $\hat{\eta}_{i+\frac{1}{2},i} = \eta_{i,i}$ ,  $\hat{u}_{i+\frac{1}{2},i} = u_{i,i}$ ,  $\hat{\vartheta}_{i+\frac{1}{2},i} = v_{i,i}$ . 若  $\eta_{i,i} = \eta_{i+1,i} = 0$ , 则取  $\hat{\eta}_{i+\frac{1}{2},i} = \hat{u}_{i+\frac{1}{2},i} = \hat{\vartheta}_{i+\frac{1}{2},i} = 0$ ,  $R_{i+\frac{1}{2},i}^1 = R_{i+\frac{1}{2},i}^2 = R_{i+\frac{1}{2},i}^3 = (0, 0, 0)^T$ ,  $L_{i+\frac{1}{2},i}^1 = L_{i+\frac{1}{2},i}^2 = L_{i+\frac{1}{2},i}^3 = (0, 0, 0)$ .

对应(2.1)中的  $\frac{dG}{dU}$  的特征值为:  $v - \sqrt{g\eta}$ ,  $v$ ,  $v + \sqrt{g\eta}$ . 对应各特征值的右特

征向量为:

$$\left( 1, \frac{u}{\eta}, -\sqrt{\frac{g}{\eta}} \right)^T, (0, 1, 0)^T, \left( 1, \frac{u}{\eta}, \sqrt{\frac{g}{\eta}} \right)^T$$

对应各特征值的左特征向量为:

$$\left(1, 0, -\sqrt{\frac{\eta}{g}}\right), \left(-\frac{u}{\eta}, 1, 0\right), \left(1, 0, \sqrt{\frac{\eta}{g}}\right)$$

依据文献[3]及参考(2.5)–(2.21)我们可以给出  $L_y$ 。

### 三、溃坝模型计算与结果讨论

先将(1.1),(2.1)式中的参数进行无量纲化处理。若将无量纲后的参数记为  $x, y, t, u, v, \eta$ , 实际有量纲参数记为  $\bar{x}, \bar{y}, \bar{t}, \bar{u}, \bar{v}, \bar{\eta}$ , 则有如下关系:

$$\bar{x} = Hx, \bar{y} = Hy, \bar{t} = \frac{H}{c_0}t, \bar{u} = c_0u, \bar{v} = c_0v, \bar{\eta} = H\eta$$

式中,  $H = 100\text{m}$ ,  $g = 9.8\text{m/s}^2$ ,  $c_0 = \sqrt{gH}$ 。

#### 1. 一维溃坝问题

一维溃坝的计算模型如图 1 所示, 将河流方向取为  $x$  轴方向, 河坝处取为坐标原点。初始条件为:

$$\bar{\eta}(\bar{t}, \bar{x})|_{\bar{t}=0} = \begin{cases} 100\text{m} & \bar{x} \leq 0 \\ 1\text{m} & \bar{x} > 0 \end{cases}$$

$$\bar{u}(\bar{t}, \bar{x})|_{\bar{t}=0} = 0\text{m/s}$$

式(1.1)中的一些参数取法如下:

$$g = 9.8\text{m/s}^2, R(\bar{u}) = g\bar{u}^2/c^2\bar{\eta}, c = 52\text{m}^{1/2}/\text{s},$$

$$\frac{\partial h}{\partial x} = 0 \quad (\text{河床取为平底})$$

从图 2 中的水深曲线可见在洪水波前沿处是一激波现象, 激波后经过一平缓的过渡区是一膨胀波; 水流速曲线形状呈直角梯形状, 这也说明了激波和膨胀波现象。

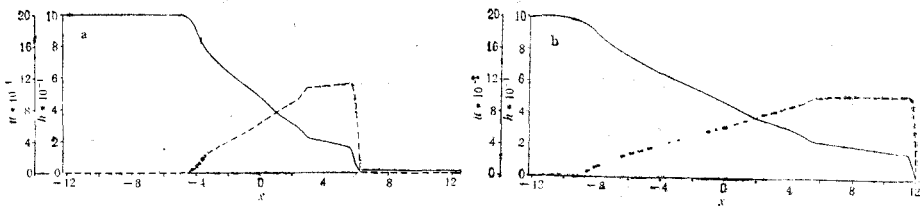


图 2 一维溃坝的数值结果图象

Fig. 2 Numerical result of one-dimensional dam-breaking

a.  $t = 4.9233$ ; b.  $t = 9.9738$ . —水深曲线; ----流速曲线。

#### 2. 二维溃坝问题

二维溃坝的计算模型为一喇叭状的河道, 河坝位于喇叭状的开口处 (见图 3)。将河流的对称中心轴向取为  $x$  轴方向, 河坝的对称中心点取为坐标原点,  $y$  轴方向为垂直河流对称轴方向。

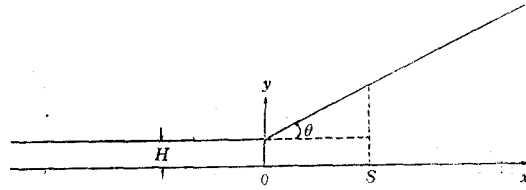


图3 二维溃坝的边界条件俯视图

( $H = 1.25\text{m}$ ,  $\text{tg}\theta = 0.5$ ; 本图给出了对称的一半)

Fig. 3 The boundary condition (vertical view) of two-dimensional dam-breaking ( $H = 1.25\text{m}$ ,  $\text{tg}\theta = 0.5$ ; only one half of symmetry)

$x$ 轴为河流对称中心,虚线与 $x$ 轴相交的 $S$ 点处为下游距河坝为5个单位处。

初始条件如下:

$$\bar{\eta}(\bar{x}, \bar{y}, 0) = \begin{cases} 100\text{m} & \bar{x} \leq 0 \\ 1\text{m} & \bar{x} > 0 \end{cases},$$

$$\bar{u}(\bar{x}, \bar{y}, 0) = \bar{v}(\bar{x}, \bar{y}, 0) = 0\text{m/s},$$

$$\text{岸边界形状为 } l: \bar{y} = \begin{cases} 125 & \bar{x} \leq 0 \\ 125 + \frac{1}{2}\bar{x} & \bar{x} > 0, \end{cases}$$

岸边界的法向速度分量取 0,  $u_n|_l = 0$ 。

式(2.1)中的一些参数取法如下:

$$g = 9.8\text{m/s}^2, \frac{\partial h}{\partial \bar{x}} = \frac{\partial h}{\partial \bar{y}} = 0 \text{ (河床取为平底),}$$

$$R_x = g\bar{u}^2/c^2\bar{\eta}, R_y = g\bar{v}^2/c^2\bar{\eta}, c = 52\text{m}^{1/2}/\text{s}。$$

在数值计算中,对岸边界作不渗透固壁边界处理,对河流对称中心作对称边界处理,所有参数进行了无量纲化处理。

从图4可以看出水深和沿河流方向的流速大小趋势在  $y = 0.5$  剖面上,与一维结果相似。不过,河流对称中心附近的水深,在前沿激波后,很快下降,呈现出水跃现象。 $x$ 方向的流速,在坝基附近很快加速,但在水跃现象部位速度下降。 $y$ 方向的流速,在坝附近和洪水波前沿处有两个峰值。

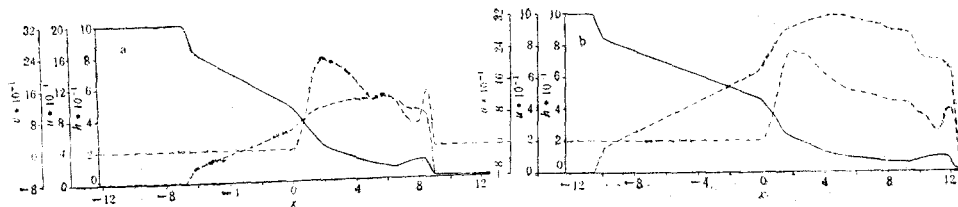


图4 二维溃坝河流  $y = 0.5$  剖面上的参数曲线

Fig. 4 Parameter curve of two-dimensional dam-breaking on the section of  $y = 0.5$

——水深曲线; --- $x$ 方向流速曲线; -·-·- $y$ 方向流速曲线。

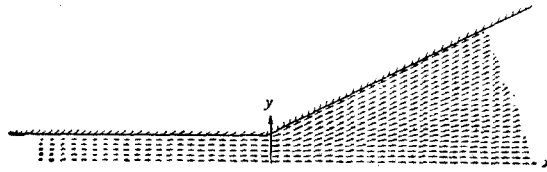
图 5 二维溃坝的流场流动图象( $t = 12.2313$ )

Fig. 5 Fluid flow of the two-dimensional dam-breaking

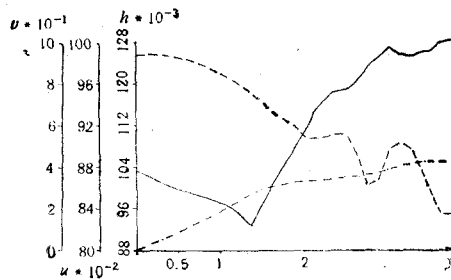
图 6 二维溃坝下游 5 个单位处横截面上的参数曲线( $t = 12.2313$ )

Fig. 6 Parameter curve on the down-stream five units' cross section of two-dimensional dam-breaking

(图例同图 4)

流场运动图象(图 5)展示了水流的运动方向和速度,流场呈扇形分布,岸边界的流向与岸界平行。

从图 6(截面位置见图 3)中的虚线位置看出,水深曲线有两个主峰,一个是河流的对称中心,另一个则在岸边,且岸边的水深比河中心还深。 $x$  方向上的流速也有两个主峰,一个是河中心,一个则靠近岸边。我们认为这些现象的产生与所取岸界为固壁边界有关。从  $x$  方向的流速曲线还可以看到,在溃坝下游横截面上有激波现象。从  $y$  方向的流速曲线可看出,由河流中心到岸界是一个逐渐增加的过程。

由于本文工作受微机的内存及速度的限制,使网格划分比较粗,故二维河道拐角处的流动模拟显得粗糙,对粘性边界及变化河床的模拟,也有待于进一步研究和探讨。

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## THE TVD FINITE DIFFERENCE SCHEME TO SIMULATE THE SHALLOW WATER WAVE EQUATIONS\*

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### ABSTRACT

The TVD (Total Variation Diminishing) finite difference method for the shallow water wave equations in one dimension and two dimensions is based on Harten's conservation form equations' TVD idea to simulate gas dynamic shock wave equations extensively and successfully.

In this paper, one and two-dimensional shallow water wave conservation form equations are given, TVD idea is applied to these conservation form equations to get the shallow water wave equations' TVD scheme.

Successful simulation was made of the shock wave and the rarefaction wave phenomena about an one-dimensional dambreaking computational model and a bugle shaped two-dimensional dam-breaking computational model by using TVD scheme. The numerical solutions of this paper have always preserved sharpness and nonoscillation near the place where the solution is discontinuous.

Different results between the two-dimensional model and the one-dimensional model are found, for example, the two-dimensional model's results show a faster descent behind the shock wave and a greater depth near the shore than that in the centre of the river.

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